CREEP STRENGTH MODEL WITH NONMONOTONIC DEPENDENCE OF THE STRAIN DURING RUPTURE ON THE STRESS

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The development of the experimental and theoretical aspects of investigations on the creep and creep strength of metals proceeds at an ever-accelerating pace. However, up to now a relatively small quantity of reliable experimental data has been accumulated which will characterize the creep of metals down to fracture in a broad range of stresses. This is explained by the serious difficulties encountered by researchers in measuring the creep strain of specimens subjected to constant stress (or load) under high temperature conditions during a long time (sometimes many thousands of hours). Consequently, such tests often do not end in rupture (i.e., the creep strength is not examined), or they result in rupture without a measure of the strain during creep. The boundedness and separateness of the actual material result in the fact that formulation of the equations describing material creep down to rupture has not taken place definitively even in the case of the uniaxial stress state.

Rabotnov's conception of a mechanical equation of state [1] with a system of kinetic equations to determine the parameters characterizing the state under consideration is most nearly complete in describing the process of the creep of structural materials. According to this conception, the creep rate $\dot{p}$ is determined by the stress $\sigma$, the temperature, and a certain quantity of structural parameters, which vary in conformity with the kinetic equations during creep. The structural parameter $\omega(t)$, which is a certain measure of the "spallation" of the material, is used most often in describing the creep strength. A value of $\omega$ from the range $0 \leqslant \omega \leqslant 1$ is ascribed to each "spallation" state; the value $\omega = 0$ here corresponds provisionally to undamaged material, and $\omega = 1$ to the presence of macroscopic cracks.

In the case in which $\omega(t)$ is a single structural parameter, the creep of a material down to rupture can be described by the following system of equations:

$$\dot{p} = f(\sigma, \omega), \quad \dot{\omega} = \varphi(\sigma, \omega). \tag{1}$$

Equations (1) in the form of simple power-law dependences

$$\dot{p} = a\sigma^n(1-\omega)^{-s}; \tag{2}$$

$$\dot{\omega} = b\sigma^k(1-\omega)^{-r} \tag{3}$$

are most often used in describing creep characterized by steady and accelerating stages. Let us examine the case of brittle fracture when the change in cross-sectional area of the specimen can be neglected down to fracture at the time t* because of the relative smallness of the strain. Setting $\sigma(t) = \text{const}$, we integrate (3)

$$\omega(t) = 1 - (1 - t/t^*)^{\left(\frac{1}{r+1}\right)}, \quad t^* = [b(1+r)\sigma^k]^{-1}. \tag{4}$$

We insert (4) into (2) and integrate

$$p(t) = \frac{a}{b(r-s+1)}\sigma^{(n-k)}\left[1 - (1 - t/t^*)^{\left(\frac{r+1-s}{r+1}\right)}\right]. \tag{5}$$

The magnitude of the strain $p(t^*) = p^*$ at the time of fracture will be, according to (5),

$$p^* = \frac{a}{b(r-s+1)}\sigma^{(n-k)}. \tag{6}$$

In conformity with (6) the deformation p* is a monotonically increasing function of the stress $\sigma$ for $n > k$ and a monotonically decreasing function for $n < k$.

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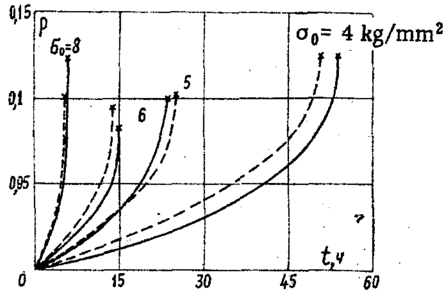

Fig. 1

The fact of a monotonic change in the quantity p* in the whole stress range is not confirmed in certain experimental investigations. Thus, it was noted in [2], in an analysis of the results of testing 15Kh1M1F steel for creep strength at 565°C in the range $10^3$ h < t* < $10^4$ h that the dependence of the creep strain at the time of fracture has a minimum at t* = 2500 h. Moreover, it is shown that the dependence of the quantity of cracks per unit surface m(t*) has a maximum that also corresponds to t* = 2500 h. A metallographic explanation of the mentioned nature of the curves p*(t*) and m(t*) is presented in the paper, and structural changes in the metal that affect the nature and quantity of fracture foci substantially are noted.

The results of tests performed at the Mechanics Institute of Moscow State University on Kh18N10T stainless steel specimens at 850°C are described in [3]. The specimens were subjected to a constant tensile force up to fracture. A total of 21 specimens of one melt were tested for four values of the initial stress $\sigma_0$:$\sigma_0$ = 4, 5, 6, and 8 kg/mm². The averaged experimental curves corresponding to each of these values of $\sigma_0$ are superposed by continuous lines in Fig. 1. The quantity of specimens N tested at each stress, as well as the mean values of t* and p* for each $\sigma_0$, are indicated in Table 1 for g = 1. It follows from Table 1 that the strain p* at the time of fracture depends nonmonotonically on $\sigma_0$: in the range investigated $\sigma_0$ has the value $\sigma_0 = \sigma_*$, for which $p^*(\sigma_*)$ is minimal. Within the framework of the relations (2) and (3) it is impossible to describe this effect. The condition n = k was used in [3] to describe these tests, hence $p^*(\sigma_0)$ = const.

Let us replace the power-law dependence of $\dot{p}$ on σ in (2) by a function of the hyperbolic sine. We obtain the fundamental system of equations (with five material constants) in the form

$$\dot{p} = a\frac{\mathrm{Sh}(\sigma/c)}{(1-\omega)^n}; \tag{7}$$

$$\dot{\omega} = b\sigma^k/(1-\omega)^k. \tag{8}$$

We obtain $\dot{\omega} \sim b\sigma^k$ at the beginning of the creep process from (8), hence k > 1 follows. We shall neglect the change in specimen cross section and will assume that σ(t) = σ = const. Let us divide the right and left sides of Eqs. (7) and (8), and let us integrate

$$\frac{p}{p^*} = 1-(1-\omega)^{(k-n+1)},\ p^* = \frac{a}{b(k-n+1)}\cdot\frac{\mathrm{Sh}(\sigma/c)}{\sigma^k}. \tag{9}$$

According to (9), a finite value of p* holds only for (k − n + 1) > 0.

Let us analyze the dependence of p* on σ. For small σ the function sh(σ/c) is equivalent to σ/c, i.e., p* decreases monotonically as σ grows. For large σ the function is sh(σ/c) ∿ (1/2)exp(σ/c), so that p* increases as σ grows. Indeed

$$\frac{dp^*}{d\sigma} = \frac{a}{b(k-n+1)}\left[\frac{\mathrm{ch}(\sigma/c)}{c\sigma^k} - \frac{k\,\mathrm{Sh}(\sigma/c)}{\sigma^{(k+1)}}\right].$$

As $\sigma \to 0$ we have $\frac{dp^*}{d\sigma} = -\frac{a(k-1)}{b(k-n+1)c}\frac{1}{\sigma^k} \to -\infty$. As $\sigma \to +\infty$ we have $\frac{dp^*}{d\sigma} \to \frac{a}{b(k-n+1)}\left(\frac{1}{c} - \frac{k}{\sigma}\right)\cdot$ $\frac{\exp(\sigma/c)}{\sigma^k} \to +\infty$. Equating dp*/dσ to zero, we obtain an equation to find the values of the stress $\sigma = \sigma_*$ for which the strain p* is minimal:

TABLE 1

| $\sigma_0$, kg/mm² | $N$ | g = 1 | | | g = 2 | | |
|---|---|---|---|---|---|---|---|
| | | $\dot{p}_0$, h⁻¹ | $t^*$, h | $p^*$ | $\dot{p}_0$, h⁻¹ | $t^*$, h | $p^*$ |
| 4 | 6 | 0,00082 | 54,0 | 0,126 | 0,00100 | 51,0 | 0,127 |
| 5 | 7 | 0,00190 | 23,5 | 0,100 | 0,00176 | 25,2 | 0,103 |
| 6 | 6 | 0,00300 | 15,4 | 0,082 | 0,00310 | 14,1 | 0,094 |
| 8 | 2 | 0,00770 | 6,0 | 0,124 | 0,00935 | 5,7 | 0,100 |

$$\mathrm{th}\left(\frac{\sigma_*}{c}\right) = \frac{1}{k}\frac{\sigma_*}{c}. \tag{10}$$

Substituting (9) into (7) and integrating, we obtain the equation of the creep curve:

$$\frac{p}{p^*} = 1 - \left(1 - \frac{t}{t^*}\right)^{\left(\frac{k-n+1}{k+1}\right)},\ t^* = [b(k+1)\sigma^k]^{-1}. \tag{11}$$

The constants in the equations of the model (7) and (8) are determined as follows. For steady creep we have $\dot{p}_0 = a\ \mathrm{sh}\ (\sigma/c)$. We determine the constants $a$ and c by least squares from the condition of correspondence between the experimental and computed values of $\dot{p}_0$. The parameters k and b are determined by least squares by using (11). Afterwards, we find the remaining constant n by taking the average of p* according to (9).

Processing the test results on Kh19N10T stainless steel at 850°C by this method results in the following values of the constants: $a = 3.12\cdot10^{-4}$ h$^{-1}$, c = 2.05 kg/mm², k = 3.17, b = $0.58\cdot10^{-4}$ (kg/mm²)$^{-3.17}$ h$^{-1}$, n = 2.36. Taking account of these quantities, the creep curves (11) are presented by dashes in Fig. 1. Theoretical values of $\dot{p}_0$, t*, and p* are presented in Table 1 (for g = 2) for each stress $p_0$, t*, and p*. According to (10) the value of σ* is 6.5 kg/mm², i.e., it is within the stress range under investigation.

Let us note that a power-law function of the stress p*(σ) can be introduced in (7) to describe the creep curves for which the dependence σ has a local maximum, and a function of the hyperbolic sine in (8).

Thus, the introduction of different functional relationships to take account of the influence of the stress on the creep rate and the cumulative damage rate affords a possibility for describing the nonmonotonic dependence of the rupture strain on the stress level.

## LITERATURE CITED


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